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GROUNDWATER CAPTURE ZONE ANALYSIS AND MODELING SIMULATIONS

AMERICAN RIVER STUDY AREA RANCHO CORDOVA, CALIFORNIA

Environmental Protection Agency Contract No. 68-W9-0031
GROUNDWATER CAPTURE
ZONE ANALYSIS AND
MODELING SIMULATIONS

AMERICAN RIVER STUDY AREA
RANCHO CORDOVA, CALIFORNIA

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June 17, 1994
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Groundwater Capture Zone
Analysis and Modeling Simulations

Purpose

The purpose of this report is to present the results of groundwater modeling simulations performed to estimate the extent of capture zones created by several proposed groundwater extraction wells in the vicinity of the American River near Rancho Cordova, California. The interpretations of aquifer tests performed at the site are also described as they relate to the refinement of the hydraulic properties assumed for input to the groundwater model.

CH2M HILL performed this effort in an oversight role to investigate the validity of the assumptions and conclusions presented in the May 1993 Engineering Evaluation/Cost Analysis (EE/CA) for the American River Study Area produced by Aerojet. It is intended as a management tool to focus future remedial activities in the American River Study Area and is not intended to be an exhaustive analysis of the hydrogeologic conditions that currently exist, or will exist in the future as a result of response actions.

A response action is necessary in the American River Study Area because groundwater contamination presents a potential threat to the long-term beneficial use of groundwater resources in the area.

Introduction

A plume of contaminated groundwater has been detected offsite from the Aerojet Facility and it is continuing to move northwesterly toward and beneath the American River. This plume is composed predominantly of trichlorethene (TCE) and it is the target of a planned response action to remediate the contaminated aquifers in the area. Current data shows that the three uppermost aquifers in the vicinity of the American River contain levels of TCE that exceed the State of California Maximum Contaminant Levels (MCLs). A description of the hydrogeologic and water quality data is available in the document titled Engineering Evaluation and Cost Analysis for the American River Study Area, produced by Aerojet General Corporation, May 1993. Additional aquifer testing data collected since May 1993 were also used in this analysis.

Conclusions and Recommendations

The main conclusions reached from the groundwater capture zone analysis and modeling simulations, and the recommended course of action, are as follows.

1. The continued migration of the American River Study Area plume is a potential threat to municipal wells of the Fair Oaks Water District (FOWD).
2. The implementation of the removal action recommended in Aerojet's May 1993 EE/CA is an environmentally sound first step in an iterative approach to control the migration of the contaminant plume in the American River Study Area.

3. A significantly higher extraction rate than discussed in the EE/CA, on the order of 2,000 gallons per minute (gpm), may be required to completely capture the TCE target area in the upper, middle, and lower aquifers. The estimated pumping rate is higher than had been contemplated in the EE/CA because the aquifers have been tested and found to be more transmissive. The model used in this analysis assumes that the aquifer properties disclosed in the testing at Well Cluster 4300-02 extend throughout the modeled area. The assumption was made because there are no other quantitative data on the properties of the upper or middle aquifers. The travel times shown on the figures, as indicated by the tic marks, do not reflect actual groundwater contaminant migration observed at the site. This discrepancy should be continually evaluated through implementation of this project.

4. An extraction well network, designed to contain the observed contamination, will require additional upper, middle, and lower aquifer wells on the north and south sides of the American River.

5. On the basis of groundwater monitoring data collected, including data from the newly installed Greenvale (Aerojet Well No. 1559-61) and Oak Glenn (Aerojet Well No. 1556-8) wells, the modeling shows that the recharge of groundwater in the proposed locations of the May 1993 EE/CA will not accelerate the identified contaminant target area toward the FOWD water supply wells.

6. Recent groundwater quality data collected from Aerojet Well No. 1407 by Aerojet and the Central Valley Regional Water Quality Control Board (RWQCB) indicate that TCE concentrations above MCLs (at 7.7 µg/l) have been detected (RWQCB Inspection Report dated April 27, 1994). These data were not received in time for inclusion in this analysis. Previous sampling rounds for Well No. 1407 have consistently produced results below MCLs. Additionally, data collected from Aerojet Well No. 1526, located between the TCE plume and Well No. 1407, have yielded samples that have been consistently "nondetect" for TCE in previous sampling rounds. Until this situation is fully evaluated, we recommend that recharge in the vicinity of Well No. 1407 be in the deeper aquifer.

7. Therefore, Aerojet should initiate action to produce future phases of groundwater extraction in the northern and southern areas of the American River.

We recommend that Aerojet implement the following course of action in the American River Study Area:

- Installation of the proposed system as identified in the May 1993 EE/CA, with the exception of an increased design flow rate.
Evaluation and implementation of extraction, treatment, and disposal of groundwater in the southern portion of the American River Study Area.

Continue monitoring and studying the actual response of the aquifer and influence on the contaminant plume in the American River Study Area from groundwater extraction. This study should be the basis for using a phased approach to address the groundwater contamination in the vicinity. It should include the collection of actual data, as well as the refining of the model with newly collected data.

All groundwater models are simplified versions of real world systems that approximately simulate the relevant reactions of a real world system. Models are only as precise as the data and assumptions upon which they are based. Because real world systems are very complex, as is the case with the American River Study Area, the modeling results presented in this report should be viewed as an exercise and as a useful tool to help facilitate removal and response action decisions.

In general, the results of this modeling exercise, when compared to the known real world movement of the contaminant plume in the American River Study Area, are not in complete agreement. Of specific concern is that the travel times and resultant flow rates are extremely high values. In reality, the historically monitored groundwater movement in the American River Study Area has not proved to be this rapid. Therefore, the actual capture of groundwater contaminants estimated in this report may differ from the actual capture zones realized in real world operations. If the actual transmissivity of the aquifers is less than what was used in the model, then less pumping from potentially more wells may be required.

Aquifer Test Evaluation

Aquifer Test Description

Several aquifer tests have been performed in Extraction Wells 4300, 4301, and 4302 which were screened specifically in the upper, middle, and lower aquifers, respectively. This well cluster, located north of the American River, is shown on Figure 1. Initially, three separate aquifer tests were conducted at the well cluster during November 1993. Each test consisted of pumping one well in the cluster for approximately 4 days, and monitoring the water level response in the adjacent extraction wells and in surrounding monitoring wells. An additional aquifer test was conducted in December 1993, consisting of pumping all three extraction wells concurrently for a period of 10 days, and monitoring the water level response in surrounding monitoring wells.
Aquifer Test Analysis

The method used to interpret the data from the two sets of aquifer tests varied depending on the test design. The single-well pumping tests were evaluated using the computer program MLU developed by C.J. Hemker, 1993. The results from the multiple well test were evaluated using the groundwater model MicroFem, also written by Hemker, et al, 1987 to 1994. A more detailed description of these analysis methods is presented below.

**Single-Well Test Analysis**

The computer program MLU was used to calculate hydraulic parameter estimates from the single-well aquifer tests. MLU is a transient, multi-aquifer simulation that uses a least squares, curve-fitting algorithm to calculate aquifer and aquitard parameters (aquifer transmissivity \([T]\), aquifer storage coefficient \([S]\), aquitard resistance \([R]\), and aquitard storage coefficient \([S']\)) on the basis of time-drawdown data collected during aquifer tests. The solution technique accounts for both leakage between aquifers and storage of water in the aquitards and aquifers. Any number of parameters can be fixed based on prior knowledge of their approximate values; the program then estimates the values of the remaining parameters.

A more complete description of this program, including the governing equations and the solution technique is presented in Appendix A. In Table 1, the results of the MLU analysis are summarized and compared to the estimates of aquifer parameters presented in the EE/CA. The EE/CA estimates were based on an aquifer test of a long-screened water supply well that was located approximately 1 mile to the southeast of short-screened Wells 4300, 4301, and 4302.

**Table 1**

<table>
<thead>
<tr>
<th></th>
<th>EE/CA Data</th>
<th>Single-Well Tests</th>
<th>Multiple-Well Tests</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>Aquifer Transmissivity (ft²/d)</strong></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Upper Aquifer</td>
<td>2,000</td>
<td>22,000*</td>
<td>16,000*</td>
</tr>
<tr>
<td>Middle Aquifer</td>
<td>550</td>
<td>1,830*</td>
<td>4,000*</td>
</tr>
<tr>
<td>Lower Aquifer</td>
<td>1,730</td>
<td>90*</td>
<td>125*</td>
</tr>
<tr>
<td><strong>Aquitard Vertical Resistance (days)</strong></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Upper/Middle</td>
<td>N/A</td>
<td>80*</td>
<td>80*</td>
</tr>
<tr>
<td>Middle/Lower</td>
<td>N/A</td>
<td>900*</td>
<td>900*</td>
</tr>
</tbody>
</table>

*Parameter Values in the Vicinity of Wells 4300, 4301, and 4302
N/A = Not available
Multiple-Well Test Analysis

Because the MLU program can evaluate only one pumped aquifer at a time, a three-dimensional groundwater flow model was used to evaluate the multiple-well test results. A finite element MicroFem groundwater model was developed by CH2M HILL to estimate the capture zones that would be produced by operating the proposed extraction wells located near the American River. This model was based on the results of the single-well tests and the stratigraphic data presented in the American River Study Area EE/CA. The results of the 10-day multiple-well aquifer test provided an additional opportunity to test the validity of the existing groundwater model, and provided another data set with which the model could be further calibrated. A more complete discussion of the groundwater model construction and development is provided in the following section.

The 10-day multiple-well aquifer test was simulated using the finite element groundwater model and, following small adjustments to the assumed aquifer transmissivities, produced a good match between observed and simulated drawdown in the vicinity of the extraction wells. The observed water levels at the end of the 10-day test were increased by 0.5 foot to approximately account for a regional rising water level trend of about 0.5 foot observed during the course of the test. Comparisons of observed and simulated drawdown at various distances from the pumping wells are shown on Figures 2 through 4. The agreement between simulated and observed drawdown improves as radial distance from the pumping well increases. Because the objective of this modeling effort is to define the extent of the capture zone created by extraction wells, it is more important to match the observed drawdown at the distant edges of the cone of depression. The aquifer parameters determined from the multiple-well aquifer test were similar to those obtained from the single-well tests. Table 1 presents the aquifer parameter estimates that are based on the multiple-well test.

Groundwater Model Construction and Calibration

The groundwater model selected to evaluate the extent of capture produced by the proposed extraction wells in the American River Study Area is MicroFem. This finite element model was selected because it is capable of simulating transient groundwater flow in multiple aquifer systems, and includes leakage between adjacent aquifers at the site. It is capable of generating three dimensional flowlines that allow for the evaluation of capture zones in three dimensions. The capability of incorporating the leakage between aquifers, and the influence of leakage on the capture of contaminated groundwater by extraction wells, was needed for the evaluation of the leaky aquifer system that exists in the study area.

Groundwater Model Assumptions and Limitations

The conclusions presented in this report are based on a groundwater model that was constructed using many assumptions regarding the distribution of aquifer properties across the American River Study Area. The uncertainty associated with our current understanding of
Figure 2
Simulated versus observed drawdown in the upper aquifer after 10 days of pumping.
Aerojet-American River Area
Rancho Cordova, California
FIGURE 3
SIMULATED VERSUS OBSERVED
DRAWDOWN IN THE MIDDLE
AQUIFER AFTER 10 DAYS OF PUMPING
AEROJET-AMERICAN RIVER AREA
RANCHO CORDOVA, CALIFORNIA
FIGURE 4
SIMULATED VERSUS OBSERVED DRAWDOWN IN THE LOWER AQUIFER AFTER 10 DAYS OF PUMPING
AEROJET-AMERICAN RIVER AREA
RANCHO CORDOVA, CALIFORNIA
the actual aquifer properties at the site is still large. As such, these predictions of aquifer response to extraction well pumping also contain significant uncertainty. The construction of any response action at the site must be associated with sufficient field information, collected during operation, to confirm that the desired extent of capture is being achieved by the system in place.

The capture zone simulations presented in this analysis were performed assuming steady-state conditions. This neglects the inclusion of transient influences on the groundwater system such as seasonal fluctuations in groundwater levels and recharge rates. However, the use of a steady-state model is appropriate because the objective of the groundwater modeling effort is to evaluate the long-term performance of an extraction system at containing and extracting contaminated groundwater.

Groundwater Model Construction

The principal objective of the model prepared for this analysis was to test the assumptions and conclusions presented in the EE/CA. Thus, CH2M HILL performed this modeling effort primarily in an oversight role. The model should not be considered to be an exhaustive analysis of the hydrogeologic conditions in the American River area. However, the model is useful for evaluation of hydrogeologic conditions in the study area to determine the need and scope of response action to control the movement of contaminated groundwater.

The groundwater model used in this analysis has evolved since September 1993, as additional field data have been collected. The original groundwater model was a five-layer model and was based entirely on the aquifer and aquitard geometries, and the aquifer properties presented in the May 1993 EE/CA Report. The upper three model layers represent the upper, middle, and lower aquifers at the site. The fourth and fifth layers combine to represent the regional deeper aquifer, which was subdivided into two layers to allow greater vertical resolution of deep flow paths. The original aquifer properties obtained from information presented in the EE/CA are summarized in Table 1.

The lateral boundary conditions of the model were defined as constant heads in all layers, and are based on water level contour maps presented in the Aerojet EE/CA (Figures 2-10 through 2-12). The upper and lower boundaries were defined as no flow boundaries.

Groundwater Model Calibration

The original calibration of the groundwater model was based on comparing the simulated groundwater levels across the study area with the observed water levels measured in monitoring wells. The calibration procedure consisted of setting the constant heads, located at the model perimeter, to produce groundwater flow directions and gradients to match the observed values. This procedure resulted in a close agreement between the simulated and the observed water levels.
Revisions to Groundwater Model

After completion of the single-well tests at Wells 4300, 4301, and 4302, the model was revised to reflect the findings from these tests. These tests suggested that the transmissivity of the upper aquifer was significantly higher than the values estimated for the EE/CA, and that the transmissivity of the lower aquifer was significantly lower. The thickness of each of the aquifer units was presented in the EE/CA (Figures 2-7 through 2-9), and the influence of aquifer thickness on transmissivity was retained in the refined version of the groundwater model. The aquifer thickness was accounted for by adjusting all transmissivity values in a single layer by a common factor. This approach is equivalent to adjusting the hydraulic conductivity of the aquifer materials while retaining the original estimates of aquifer thickness. The leakance values assigned between model layers were based on the results of the MLU analysis described above (see Table 1).

The material properties assigned to the groundwater model were further refined on the basis of the 10-day aquifer test conducted at the site. As previously discussed, the 10-day multiple-well test was simulated using the MicroFem groundwater model. Following small adjustments to the assumed aquifer transmissivities, a good match between the observed and the simulated drawdown in the vicinity of the extraction wells was obtained (see Figures 2 through 4). The results of this calibration effort were critical because the purpose of constructing the groundwater model was to evaluate the response of the aquifer system to proposed extraction well networks. These results suggest that the groundwater model produces simulated water levels in response to pumping that are close to those that will be observed in the vicinity of Wells 4300, 4301, and 4302.

However, in areas remote from the Well 4300 series, where little field data are available, model predictions are less certain. As the capture wellfield is constructed, a program of aquifer testing and model refinement should be performed to enhance the predictive capability of the model.

Groundwater Capture Evaluation

The following groundwater capture evaluation considers two potential wellfields designed to remediate contaminated groundwater in the vicinity of the American River. Wellfield 1 was proposed by Aerojet in the EE/CA, and consisted of two extraction well clusters pumping at the rates presented in Table 2. The Aerojet wellfield was based on the results of the two-dimensional model, Flowpath, which relied on aquifer parameters from a well located approximately 1 mile to the southeast of Wells 4300-4302. The extraction rates in Table 2 were based on the pumping tests in November and December 1993, and not on the assumed extraction rates in the EE/CA.

Wellfield 2 was designed by CH2M HILL to capture the entire plume of contaminated groundwater north of the American River. The extraction rates for this wellfield are also listed in Table 2.
### Table 2

**Assumed Extraction Rates for the Groundwater Modeling Simulations**

<table>
<thead>
<tr>
<th>Well Name</th>
<th>Upper Aquifer</th>
<th>Middle Aquifer</th>
<th>Lower Aquifer</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>EE/CA Wellfield 1</strong></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Wells 4300 to 4302</td>
<td>224</td>
<td>197</td>
<td>34</td>
</tr>
<tr>
<td>Cluster 1</td>
<td>224</td>
<td>197</td>
<td>34</td>
</tr>
<tr>
<td>Total Wellfield Pumpage</td>
<td></td>
<td></td>
<td>910</td>
</tr>
<tr>
<td><strong>Wellfield 2</strong></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Wells 4300 to 4302</td>
<td>285</td>
<td>195</td>
<td>35</td>
</tr>
<tr>
<td>Cluster 1</td>
<td>240</td>
<td>200</td>
<td>35</td>
</tr>
<tr>
<td>Cluster 2</td>
<td>240</td>
<td>105</td>
<td>35</td>
</tr>
<tr>
<td>Cluster 3</td>
<td>235</td>
<td>105</td>
<td>35</td>
</tr>
<tr>
<td>Upper Well 4</td>
<td>285</td>
<td>0</td>
<td>0</td>
</tr>
<tr>
<td>Total Wellfield Pumpage</td>
<td></td>
<td></td>
<td>2,030</td>
</tr>
</tbody>
</table>

Notes: All Values in gpm  
Well Locations Shown on Figures 5 through 7 and 9 through 11

---

**Wellfield 1 (Wellfield Proposed by Aerojet in EE/CA)**

**Configuration and Extraction Rates**

The original extraction network proposed by Aerojet consisted of two extraction well clusters located on the downgradient edge of the TCE plumes. The locations of these well clusters are shown on Figure 5. Wells 4300 through 4302 have been constructed and were used to define aquifer parameters. The pumping rates for the second well cluster were assumed to be equal to the rates of Wells 4300 through 4302 during the 10-day aquifer test. Because all three wells will be pumping when the final extraction network is in operation, these are appropriate pumping rates for use in the capture zone simulation. The pumping rate is a combined 455 gpm from the well cluster as follows: 224 gpm from the upper aquifer, 197 gpm from the middle aquifer, and 34 gpm from the lower aquifer. The assigned pumping rates used in the capture zone simulations are summarized in Table 2.

**Capture Zone Extent**

The results of these simulations suggest that the extraction well configuration proposed by Aerojet will need to be augmented by additional extraction wells to capture the TCE plume north of the American River in all three contaminated aquifers. The extraction well locations, and the groundwater pathlines indicating the extent of capture from these wells for the upper, middle, and lower aquifers, are shown on Figures 5 through 7. The significant difference between the capture zones presented in the EE/CA Report and those presented here is mainly a result of the differences in the assumed transmissivities of the aquifer units. The transmissivity of the upper aquifer used in the EE/CA analysis is eight to ten times lower than the value used in this analysis, and the transmissivity of the middle aquifer used
NOTES:
Tick marks represent 1 year travel time.

FIGURE 6
SIMULATED FLOWLINES
FOR WELLFIELD 1
MIDDLE AQUIFER
RANCHO CORDOVA, CALIFORNIA
AEROJET - AMERICAN RIVER AREA
in the EE/CA analysis is almost four times lower than the value used here (See Table 1). The lower assumed transmissivities used in the EE/CA resulted in much larger capture zones being predicted with similar assumed extraction rates.

**Treated Groundwater Recharge**

For the purposes of this analysis, the recharge of treated groundwater was assumed to occur to the deeper regional aquifer beneath the contaminated aquifers at the site. The results of the simulations suggest that the recharge of treated groundwater has no significant impact on the extent of capture achieved by the extraction wells. In all of the simulations performed, the quantity of recharge water was equal to the total extraction rate from the extraction wells; 910 gpm for Wellfield 1, and 2,030 gpm for Wellfield 2. The locations of the recharge wells are shown on Figures 5 through 12.

**Impacts on Town and Chicago Wells**

Another objective of this analysis was to investigate the potential impacts of the planned response action on nearby groundwater production wells. Two FOWD wells are located downgradient of the TCE target area in the American River Study Area. These wells are known as the Town Well and the Chicago Well, and their locations are shown on Figure 1. Well logs obtained for both the Town and Chicago Wells indicate that the total depth of each well is 600 feet. These wells produce water from deeper regional aquifers, below the contaminated portions of the upper, middle, and lower aquifers.

FOWD records indicate that these wells are currently operated only in peak demand periods during the summer months. Total pumpage from the Town Well was 0, 3.1 and 1.0 acre-feet (ac-ft) in 1990, 1991, and 1992, respectively. Total pumpage from the Chicago Well was 9.0, 3.7, and 1.2 ac-ft in 1990, 1991, and 1992 respectively. These production quantities correspond to average pumping rates of between 0 and 5.6 gpm assuming continuous pumping throughout the year. In actuality, the wells were pumped at significantly higher rates for limited durations of time. Actual 1990 through 1992 production rates for the Town and Chicago Wells are not available. Because this analysis was performed assuming steady state conditions, a constant pumping rate had to be assigned to the production wells. The pumping rates assigned to these wells was 600 gpm for the Chicago Well and 1,050 gpm for the Town Well. These values were obtained from well testing data collected at the time of well construction.

We concluded that for the purposes of this analysis it was more appropriate to simulate the hydraulic conditions that will actually result from the operation of these wells than use an artificially low average annual pumping rate. The groundwater pathlines that provide water to both the Town Well and the Chicago Well are shown in Figure 8. It should be noted that this figure is based on conservative assumptions and most of the time the Town and Chicago Wells will be shut down.
LEGEND

- Wall location showing well number
- Recharge Well
- Upper Aquifer
- Middle Aquifer
- Lower Aquifer
- Deep Aquifer

EXTENT OF TCE TARGET AREA
IN THE LOWER AQUIFER
(Recharge occurs below the contaminated area)

NOTES:
Tick marks represent 5 year travel times.

FIGURE 8
TOWN AND CHICAGO WELL FLOWLINES - DEEP RECHARGE FOR WELLFIELD 1
RANCHO CORDOVA, CALIFORNIA
ACROJET - AMERICAN RIVER AREA
Wellfield 2

Configuration and Extraction Rates

Because Wellfield 1 proposed by Aerojet in the EE/CA will not likely capture the TCE plume, the model was used to develop a groundwater extraction network that was capable of capturing the TCE plume that exists in the aquifers north of the American River. None of the extraction networks evaluated here are intended to address contamination that resides in aquifers south of the American River. These zones of contamination will be addressed in future response actions.

The configuration of the extraction network required to capture the TCE plume is shown on Figures 9 through 11. This extraction network consists of four well clusters along with one additional upper aquifer well, and pumps a total of 2,030 gpm; including 1,285 gpm from the upper aquifer (240 to 285 gpm per well), 605 gpm from the middle aquifer (100 to 200 gpm per well), and 140 gpm from the lower aquifer (35 gpm per well). The assumed pumping rates in this capture zone simulation are summarized in Table 2. Because of the low transmissivity of the lower aquifer, much of the groundwater flowing into the extraction wells screened in the lower aquifer originates from deeper units (see Figure 11). However, the analysis indicates that the majority of the lower aquifer contamination is pulled toward, and eventually would be removed by the proposed well network.

Small gaps in the modeled capture zones still persist but are not deemed significant within the context of this report. In the model such gaps could be filled by increased pumping from the area between Well Cluster 4300-02 and Cluster 2. Such fine tuning of the capture zone was not performed for this evaluation because there are still large uncertainties in the assumed aquifer properties, and the precision of the existing groundwater model cannot definitively determine whether these small gaps in the capture zones will actually exist once the system is installed. Field measurements should be performed during the construction and operation of the extraction system to confirm that the extraction system is actually capturing the desired target area.

Treated Groundwater Recharge

Associated recharge of the total extraction rate from Wellfield 2 (2,030 gpm) was simulated at the reinjection location proposed by Aerojet in the western portion of Sailor Bar Park. The locations of these recharge wells are shown on Figure 12.

Impacts on Town and Chicago Wells

The impacts of Wellfield 2 on downgradient production wells were evaluated using the groundwater model. The flowlines associated with the Town Well and Chicago Well under
**NOTES:**
Tick marks represent 1 year travel times.
NOTES:
Tick marks represent 1 year travel times.
NOTES:
Tick marks represent 1 year travel times.
Wellfield 2 conditions are presented on Figure 12. As in the previous recharge simulation, most of the water pumped from the Town Well appears to originate from the reinjection wellfield.

Work Cited


Appendix A

Theoretical Basis for the MLU Program Code
UNSTEADY FLOW TO WELLS IN LAYERED AND FISSURED AQUIFER SYSTEMS

C.J. HEMKER and C. MAAS

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ABSTRACT


A solution has been developed for the calculation of drawdowns in leaky and confined multi-aquifer systems, pumped by a well of constant discharge penetrating one or more of the aquifers. In contrast to earlier solutions the effects of elastic storage in separating and bounding aquitards have now completely been accounted for.

The computing technique is based on the numerical inversion of the Laplace transform. Two different methods are used and results are compared with an analytical solution. Both Stehfest's algorithm and Schapery's least squares method yield accurate results in a fraction of the computation time required for the analytical evaluation.

Selected sets of time-drawdown and distance-drawdown curves are plotted to illustrate multiple-aquifer well flow and to compare new solutions with results which were previously published. The analogy with flow in unconfined and fissured aquifers is demonstrated by multilayer models, representing multiple-porosity formations with linear and diffusive crossflow.

INTRODUCTION

Practical groundwater investigations often show that an entire system of interconnected, more or less permeable layers responds to the withdrawal of water from one of its components. Some hydrogeological environments may be really complex indeed, but it is frequently found that these systems can be characterized as a sequence of alternating aquifers and aquitards of relatively large horizontal extent. In order to analyse the drawdown distribution caused by pumping wells in such leaky multiple-aquifer systems, several analytical solutions have been recently developed (Hemker, 1984, 1985; Hunt, 1985; Maas, 1986). The resulting analytical expressions may appear simple in matrix notation, but especially transient flow solutions require much computational effort when drawdowns are evaluated with some precision.

1Stehfest, 1970.
2Schapery, 1962.
Computer programs for the estimation of hydraulic characteristics from aquifer-test data generally take hundreds or thousands of function evaluations, which renders the application of analytical solutions for transient multi-aquifer flow less suitable. Confronted with this practical problem, both authors found a solution by turning to the technique of numerical inverse Laplace transformation. Since different inversion methods were used, viz. Schapery’s least squares (1962) and Stehfest (1970), this provided a good opportunity for comparing their performances, in particular the accuracy of results.

Another important advantage of the numerical inversion technique is that the effects of elastic storage of the aquitards are now no longer neglected but can be fully incorporated in the mathematical formulation. In this respect the presented solution can be regarded as the n-aquifer generalization of the single and two-aquifer solutions given by Hantush (1960) and Neuman and Witherspoon (1969a) respectively. The applicability of the presented n-layer transient flow solution is further enhanced by demonstrating how well flow problems of double and multiple-porosity formations can be treated in a likewise manner.
STATEMENT OF THE PROBLEM

The objective can be stated as finding a relatively fast and sufficiently accurate computing method to determine the drawdown $s$ in all the aquifers of a leaky multiple-aquifer system as a function of time $t$ and distance to a pumped well $r$. The aquifer system consists of $n$ aquifers and $n + 1$ aquitards, as shown schematically in Fig. 1. Actually this diagram represents different systems, as both no-drawdown and impervious (no-flow) boundaries are considered at the top and (or) base of the system. All aquifers and aquitards are horizontal and of infinite extent, homogeneous, isotropic and individually compressible. The well is of infinitesimal radius, fully penetrates one or more of the aquifers and discharges at a constant rate $Q$. It is further assumed that the system layers show sufficiently contrasting conductivities to neglect horizontal flow in the aquitards and resistance to vertical flow in the aquifers (Hemker, 1985).

The multi-aquifer well flow problem can now be formulated by a system of $2n$ simultaneous partial-differential equations with initial and boundary conditions for the unknown drawdown. When the radial flow component in the aquifers is considered, $s(r, t)$ must satisfy the equation:

$$
\frac{\partial^2 s_i}{\partial r^2} + \frac{1}{r} \frac{\partial s_i}{\partial r} = - \frac{K_i}{T_i} \frac{\partial s_i}{\partial z} + \frac{K_{i+1}}{T_{i+1}} \frac{\partial s_{i+1}}{\partial z} - \frac{S_i}{T_i} \frac{\partial s_i}{\partial t},
$$

$$
i = 1, 2, \ldots, n
$$

$$
s_i(r, 0) = 0
$$

$$
s_i(\infty, t) = 0
$$
while vertical flow in the aquitards is governed by:

\[ \frac{\partial^2 s'_i}{\partial z^2} = \frac{S_i}{K_i} \frac{\partial s'_i}{\partial t}, \quad i = 1, 2, \ldots, n \]  \hspace{1cm} (2)

\[ s'_i(r, z, 0) = 0 \]
\[ s'_i(r, z_{i-1}, t) = s_{i-1}(r, t) \]
\[ s'_i(r, z'_i, t) = s_i(r, t) \]

where \( K \) = hydraulic conductivity; \( S_s \) = specific storage; \( T \) = aquifer transmissivity; \( S \) = storage coefficient, and indices are used to indicate the succession of layers, while primes refer to aquitards (Figs. 1 and 5).

When a leaky system is considered \( s_o = 0 \) and \( s_{o+1} = 0 \), but if top and (or) base of the system are impervious, no-flow system boundary conditions must be used instead:

\[ \frac{\partial}{\partial z} s'_i(r, z_o, t) = 0 \]  \hspace{1cm} (3)

and (or):

\[ \frac{\partial}{\partial z} s'_{i+1}(r, z'_{o+1}, t) = 0. \]

SOLUTION IN THE LAPLACE DOMAIN

With the aid of Laplace transformation, the systems of partial differential eqns. (1) and (2) can be reduced to systems of ordinary differential equations, which may be treated by appropriate analytical techniques to find a solution in the Laplace domain. Each author independently developed an answer to the same problem. Although the methods are essentially the same, they differ in approach, notation and final formulation. In terms of matrix functions the solution may be written (Appendix A):

\[ \bar{s}(r, p) = \frac{1}{2\pi p} \bar{K}_0 [r/\bar{A}(p)] T^{-1} q \]  \hspace{1cm} (4)

which is equivalent to the result of eigenvalue analysis (Appendix B):

\[ \bar{s}(r, p) = \frac{1}{2\pi p} VKV^T q \]  \hspace{1cm} (5)

The calculation of the Laplace transformation of the drawdown vector \( \bar{s} \) is based on the construction of the tridiagonal system matrix \( A \), which is defined by:
where: 

\[ d_i = \frac{pS_i}{T_i} \]

\[ e_i = (b_i \coth b_i) e_i T_i \]

\[ f_i = b_i ((e_i, \sinh b_i) b_i = a_i D_i \]

and 

\[ a_i = \left( \frac{pS_i}{K_i} \right)^{m} \]

Diagonal matrix \( T \) contains the transmissivities, while \( q \) is simply the discharge vector \((Q_1, Q_2, \ldots, Q_n)^T\). To calculate the square matrices \( V \) and \( K \), the system matrix is first transformed into a symmetric matrix \( D \) and then decomposed into its eigenvalues \( \omega_i \) and normalized eigenvectors \( r_i \): 

\[ T = ^T A T = D = R W R^{-1} \]

Diagonal matrix \( K \) is given by its non-zero elements \( K_{ii} = (r_i \sqrt{\omega_i}) \) and matrix \( V \) is defined by \( V = T^{-m} R \). More details are given with the derivation of eqns. (4) and (5) in Appendices A and B.

**NUMERICAL INVERSION OF THE LAPLACE TRANSFORMED SOLUTION**

As it is the purpose of the authors to obtain a formula which is easy to evaluate, analytical inversion of the Laplace transformed solution has not been attempted. A large number of different numerical inversion techniques are available at present, suggesting that no single method suits all purposes. A bibliography listing over 260 titles on the subject has been compiled by Piessens (1975). Systematic comparative studies were conducted by Cost (1964) and by Davies and Martin (1979). The latter paper has the widest scope, but the former should be of interest to the hydrologist too, as the type of problem investigated by Cost shows similarities to the hydraulic head problem of groundwater flow.

Davies and Martin suggest that more than one method should be used on any unknown function, because every method breaks down on some functions.

Undoubtedly, the easiest method of numerical Laplace inversion is due to Ter Haar (1951) and Schapery (1962). It states that:

\[ f(t) \approx \left[ pf(p) \right]_{p = i0} \]

(7)

Ter Haar chooses \( \alpha = 1 \), while Schapery proposes \( \alpha = 0.5 \) (more properly: \( \alpha = \exp(-\gamma) \), \( \gamma = \) Euler's number = 0.57721...). It is readily verified that Ter Haar's method is exact when:

\[ f(t) = at + b \]

(8)

where \( a \) and \( b \) are arbitrary constants, while Schapery's method is exact when:

\[ f(t) = a \ln t + b \]

(9)
For the methods to be applicable it turns out that the conditions (8) or (9) have to be satisfied only approximately in a certain interval around $t$.

Sternberg (1969), Brutsaert and Corapcioglu (1976) and Barents (1982) used Schapery's method to solve problems that are closely related to the problem of the present paper. The method indeed yields good results in many practical cases. For the problem at hand, however, deviations were found for small values of time, which were considered to be unacceptable by the authors.

Two other numerical inversion methods are selected, viz. a second method by Schapery (1962) (known as Schapery's least squares), and the algorithm presented by Stehfest (1970). In both cases evaluation of the transformed function is required in the real $p$-domain only. Stehfest's method is found by Davies and Martin (1979) to be accurate on a fairly wide range of functions. Schapery's least squares method, on the other hand, is stated to be rarely accurate. Nevertheless Cost (1964) finds the latter method to be very promising, at least for viscoelastic stress analysis.

In this paragraph both methods are briefly described. Their performances are shown for an arbitrarily chosen testcase and the results are compared with a recently developed analytical solution (Maas, 1987).

The numerical inversion method by Stehfest requires a fixed number $N$ of $\tilde{f}(p)$ evaluations for each value of $t$. The approximate value $f^*(t)$ at $t$ of the function $f(t)$ is obtained by:

$$f(t) \approx f^*(t) = \frac{\ln 2}{i} \sum_{i=1}^{N} v_i f^\prime \left(\frac{i \ln 2}{t}\right)$$

where $N$ must be an even number. The weighting coefficients $v_i$ depend on $N$ only and need to be calculated once:

$$v_i = (-1)^{-N/2} \sum_{k=INT[(-i+1)/2]}^{MIN}\frac{k^{N/2}(2k)!((N/2 - k)!k!(k-1)!(i-k)!(2k-i)!}{i!}$$

An optimal value for $N$ should be chosen, as it depends on the type of function to be evaluated and the precision with which this is done (eigenvalues, hyperbolic and Bessel functions were computed with a relative accuracy of at least twelve digits). Greater values for $N$ improve the result theoretically, but rounding errors limit this value in practice. Calculations with different values for $N$ therefore form an essential part of testing this method.


Schapery's least squares method assumes the original function $f(t)$ to be well approximated by $m$ exponential terms:

$$f(t) \approx f^s(t) = \sum_{i=1}^{m} g_i e^{-\alpha_i t}$$

where $g_i$ and $\alpha_i$ are parameters. After choosing the $\alpha_i$ judiciously the $g_i$ are determined by minimizing the total square error $E^2$ between $f(t)$ and $f^*(t)$, given by:
$$E^2 = \int_0^\infty [f(t) - f^*(t)]^2 \, dt$$

so:

$$\frac{\partial E^2}{\partial g_i} = \int_0^\infty 2[f(t) - f^*(t)]e^{-u_i t} \, dt = 0$$

or:

$$[f(p)]_{p=1m} = [f^*(p)]_{p=1m}, \quad (i = 1, m)$$

In other words; in order to minimize $E^2$, $f(p)$ and $f^*(p)$ should coincide in $m$ points. In view of eqn. (12):

$$f^*(p) = \sum_{j=1}^m \frac{1}{\beta_j} \frac{1}{p + 1/\alpha_j}$$

Fig. 2. Time-drawdown curves for two-aquifer systems with different top and base conditions.
TABLE 1
Stehfest's algorithm tested against an analytical solution of the problem shown in Fig. 2, model D

<table>
<thead>
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<th></th>
<th></th>
<th></th>
<th></th>
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<td>s_2 (m)</td>
<td>s_1 (m)</td>
<td>s_2 (m)</td>
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</tbody>
</table>

Choosing m values α_i and equating:

\[ f^\ast \left( \frac{1}{\alpha_i} \right) - \sum_{j=1}^{m} g_j \frac{1}{\alpha_i + 1/\alpha_j} \]  \quad (i = 1, m)

So:

\[ f^\ast \left( \frac{1}{\alpha_i} \right) = \sum_{j=1}^{m} g_j \frac{1}{\alpha_i + 1/\alpha_j} \]  \quad (i = 1, m)

Choosing m values α_i and equating:

\[ f^\ast \left( \frac{1}{\alpha_i} \right) = \sum_{j=1}^{m} g_j \frac{1}{\alpha_i + 1/\alpha_j} \]  \quad (i = 1, m)

A system of m linear equations is obtained, which can be solved for g_j.

From eqn. (12) it appears that m evaluations of f(p) suffice to calculate f^\ast(t) for any value of t. Schapery makes plausible (and it is confirmed by our experiments) that f^\ast(t) tends to f(t) as m tends to infinity.

From the form of eqn. (12) it follows that Schapery's method, when applied to well hydraulics, is better suited for the recovery period than for the drawdown period. For this reason it is profitable to subtract the available steady-state solution from eqn. (3) before inversion.

For the geohydrological scheme depicted in Fig. 2, model D, results of Stehfest's and Schapery's algorithms are presented for a selected number of time values (Tables 1 and 2). The tables also display the results obtained with an analytical formula that was derived using a generalized Fourier transform technique (Maas, 1987). It appears from the tables that the number of function-
al evaluations (N and m, respectively) are decisive for the accuracy of the results. The authors suggest that for practical purposes Stehfest's method be used with N = 10 and that of Schapery with m = 100. Which of the two algorithms is to be preferred depends primarily on the number of time values for which the original function \( f(t) \) is required. Roughly speaking the two algorithms are competitive when this number equals ten. It is safe, however, to use both methods together and compare the outcome. From a computational point of view the analytical solution shown in the last columns of Tables 1 and 2 is far inferior to the approximate ones, especially for small values of time. It should not be employed unless high accuracy is necessary for theoretical reasons.

### EXAMPLES OF WELL FLOW IN MULTIPLE AQUIFERS

The computing technique described in the previous sections can be used to analyse the drawdown behaviour of simple or rather complex multiple-aquifer systems with leaky or confined boundaries. However, unconfined and heterogeneous aquifer problems can also be treated, as will be discussed in the next section. Calculations for all examples given are carried out by the same interactive program, written in double precision GW-BASIC, compiled and run on an Olivetti M24 microcomputer. Processing speed appeared to be no serious

### Table 2

Schapery's method tested against an analytical solution of the problem shown in Fig. 2, model D

<table>
<thead>
<tr>
<th>Time (days)</th>
<th>Schapery ( m = 25 )</th>
<th>Schapery ( m = 50 )</th>
<th>Schapery ( m = 100 )</th>
<th>Analytical solution [Maas, 1987, eqn. (26)]</th>
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<td>4.3178</td>
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Fig. 3. Development of the piezometric depressions in the pumped and top aquifer of a leaky four-aquifer system.

limitation since, for example, the computation of 400 drawdowns in a four-aquifer system (Fig. 3) only required 8 min.

As a first example the two-aquifer and interjacent aquitard system, used by Neuman and Witherspoon (1969b) to show the effects of unpumped aquifers and aquitard storage, is modified to include aquitards at the top and base of both aquifers. The original time-drawdown curves were obtained in two different ways: a leaky system with high hydraulic resistances of the upper and lower aquitard and a confined system with zero-storage of these aquitards produced the same results (Fig. 2, model A1 and A2). Alternative pairs of drawdown curves were computed for three cases, all with identical characteristics for both aquifers and aquitards, but differing with respect to the applied boundary conditions (Fig. 2, model B, C and D). Upper and lower aquifer drawdowns are found to be considerably reduced by these alterations, showing the effect of each individual adjustment to the model.

For a better comparison with published results and to allow any set of consistent units, the use of dimensionless parameters may be preferred. Since the present solution only assumes horizontal flow in aquifers and vertical flow in aquitards, a selected set of dimensionless parameters should be based on the related hydraulic characteristics: T, S and c, S', respectively. In a multiple-aquifer system comprising n aquifers and (n + 1) aquitards dimensionless drawdown sD can be expressed as a function of (4n + 1) dimensionless parameters: sD = dimensionless time for the pumped kth aquifer \( (T_k s_i)^{1/2} \), 2n dimensionless leakage parameters \( (r^2/T_i c_k)^{1/2} \) and \( (r^2/T_i c_{ki})^{1/2} \) and 2n dimensionless storage parameters \( S_i/S_h \) (i \( \neq k \)) and \( S_i/S_w \), where sD is defined by sD = \( 4\pi T_i s/Q_a \). Apart from these parameters the top and base conditions of the
system should be given as no-drawdown or no-flow boundaries. Although the
drawdown curves in Fig. 2 are given on a dimensionless scale, actual computa-
tion has been carried out using the parameters indicated in the lower part of
this figure.

A second example to illustrate the application of the presented solution
technique concerns the visualization of a developing cone of depression around
a pumped well in the second aquifer of a leaky four-aquifer system (Fig. 3). It
represents the fresh and brackish groundwater system in the polder area
around a well field at Lexmond in The Netherlands (Hemker, 1984). The used
set of hydraulic parameters is only partly based on aquifer test results; especi-
ally storativity values are estimates only. The induced potentiometric depres-
sion in the upper aquifer is shown in the same figure, clearly demonstrating its
delayed development.

EXTENSION TO FISSURED AQUIFER SYSTEMS

As explained by Streltsova-Adams (1978) and Streltsova (1982, 1984) there is
an obvious similarity between well flow solutions for layered systems and
fissured formations. Hence it is of interest to determine to what extent the
applicability of the presented multiple-aquifer technique can be expanded to
include the related solutions for heterogeneous systems. Most solutions for
groundwater flow in fissured formations are based on the double-porosity
concept and assume either a linear (also referred to as pseudo-steady state,
lumped) or a diffusive (transient, capacitive, distributed) type of block-to-fis-
sure flow behaviour (Streltsova, 1984; Barker, 1985). Linear crossflow implies
that the rate of flow is proportional to the average difference in heads between
fissures and blocks, while diffusive crossflow is based on the development of a
head distribution in the matrix material of the blocks.

The influence of double-porosity with linear crossflow can be simulated by
adding a zero-transmissivity aquifer and an interjacent zero-storativity aquit-
ard to the system. Unconfined conditions of multiple-aquifer systems can be
modelled in exactly the same way (Hemker, 1985). When, for example, an
unconfined fissured aquifer is considered, drawdown behaviour can be re-
produced by a confined three-aquifer model. A quantitative confirmation was
sought in the type curves presented by Boulton and Streltsova-Adams (1978).
However, although five parameters are given for each set of curves, while four
are sufficient, not enough information is provided for a reconstruction. With
the aid of published drawdown values for the same curves (Streltsova-Adams,
1978) it was found that the unknown aquifer transmissivity is probably
1 m² day⁻¹. Table 3 shows the results of calculations obtained from the analyti-
cal solution for unconfined flow in a fissured formation, the analytical solution
for confined flow in a three-aquifer system and the corresponding Stehfest
numerical inversion approximation.

When the block geometry of a double-porosity formation can be idealized as
infinite slabs, solutions for diffusive block-to-fissure flow are identical to an
TABLE 3

Drawdown values for comparable triple-porosity models

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^2 Hemker, 1985: Eqn. (11), r(T_1 c_1)^1/2 = 0.1, r(T_2 c_1)^1/2 = 1, S_a/S_a = 100, S_a/S_2 = 10.
^3 Using Stehfest (N = 10); see Fig. 4, model A, r = 1 m.

Aquifer-aquitard system with no-flow boundaries. Similarly, triple-porosity media with diffusive flow and slab-shaped blocks correspond with confined layered systems of two aquitards and one aquifer. To illustrate these analogies, the multilayer counterparts of: (A) an unconfined double-porosity formation with linear crossflow; (B) an unconfined double-porosity formation with diffusive crossflow; and (C) a confined triple-porosity formation with diffusive crossflow are shown in Fig. 4. For each model dimensionless drawdown curves are plotted to demonstrate their different hydraulic behaviour.

Thus far only double- and triple-porosity single formations have been discussed. There is, however, no restriction to also include multiple-porosity layered formations. The solution is found in the composition of the proper system matrix. To include diffusive crossflow in some layer, the corresponding diagonal element of the system matrix is extended with a source term to account for contribution from the matrix blocks: (b_j tanh b_j) c_j T_j. Two extra block parameters are introduced (c_j, S_j) when one such term is added; in case of multiple-porosity the procedure of adding terms can be repeated.

Linear crossflow can be included in a multilayer model in a similar way by adding the term b_j^2/[c_j T_j (1 + b_j^2)] to the appropriate diagonal element. If the matrix block drawdown is of interest an alternative solution is found by increasing the number of aquifers considered. The block storativity value is
assigned to an aquifer of (nearly) zero-transmissivity, while the additional zero-storativity aquitard accounts for the block resistance (Fig. 4, model A). If the fissured formation is no top or base aquifer, however, the simple structure of aquifer succession is broken. As a result the tridiagonal property of the system matrix will be lost, but this doesn't impede the application of the presented matrix solution method.

CONCLUSIONS

The main object of this paper has been to develop an efficient solution to the transient multiple-aquifer well flow problem, accounting for elastic storage in aquifers as well as in aquitards. Following essentially the same techniques, the authors independently arrived at comparable results, using the Laplace transform and numerical Laplace inversion. The two approaches prove to be competitive from an economical point of view. Being essentially approximate, the methods are recommended to be used together in order to check each other. A comparison of the results with those obtained by a fully analytical solution shows that high accuracy is attainable.

It has been shown by examples that the solutions are well suited to theoretically investigate the dynamical behaviour of multiple-aquifer systems. Features such as double or multiple porosity are easily incorporated in the models.

The primary stimulus of the research reported in this paper has been its application to computerized evaluation of pumping tests. Multiple-aquifer tests have been successfully interpreted by both authors. The subject is felt to be of sufficient practical interest to justify publication as a separate paper.
APPENDIX A. DERIVATION OF EQN. (4) IN TERMS OF MATRIX FUNCTIONS

The reader who is not familiar with the use of matrix differential calculus is advised to consult Maas (1986). A stratified porous medium is considered, containing an infinite number of layers. The following (infinite) matrices are defined:

(a) column matrices:
\[ s \begin{bmatrix} u_i \\ \end{bmatrix} = u_i \]
\[ s' \begin{bmatrix} u'_i \\ \end{bmatrix} = u'_i \]
\[ q \begin{bmatrix} q_i \\ \end{bmatrix} = q_i \]

(b) diagonal matrices:
\[ K \begin{bmatrix} k_i \\ \end{bmatrix} = k_i \]
\[ K' \begin{bmatrix} k'_i \\ \end{bmatrix} = k'_i \]
\[ S \begin{bmatrix} s_i \\ \end{bmatrix} = s_i \]
\[ S' \begin{bmatrix} s'_i \\ \end{bmatrix} = s'_i \]
\[ D' \begin{bmatrix} d'_i \\ \end{bmatrix} = d'_i \]
\[ C \begin{bmatrix} c_i \\ \end{bmatrix} = c_i \]
\[ T \begin{bmatrix} t_i \\ \end{bmatrix} = t_i \]

(c) superdiagonal matrix:
\[ H \begin{bmatrix} \ldots \end{bmatrix} = H \]

The matrix \( H \), being introduced for operational purposes, is defined to have the following property:
\[ H^T H = I \]

where \( H^T \) is the transpose of \( H \) and \( I \) is the infinite unit matrix.

The matrix differential equation describing the hydraulic head in the aquitards reads:
\[ K' \frac{\partial^2}{\partial z^2} s'(r, z, t) = S' \frac{\partial}{\partial z} s'(r, z, t) \]  

(A1)

Putting \( \zeta = (z - z)/D' \), for each aquitard, eqn. (A1) can be written:
\[ \frac{\partial^2}{\partial \zeta^2} s'(r, \zeta, t) = CS' \frac{\partial}{\partial \zeta} s'(r, \zeta, t) \]  

(A1')

(notice that \( \zeta \) runs from 0 to 1). The boundary conditions with respect to \( \zeta \) are given by:
\[ s'(r, 1, t) = Hs'(r, 0, t) = s(r, t) \]  

(A2a)
\[ C^{-1} \frac{\partial}{\partial \zeta} s'(r, \zeta, 1, t) = HC^{-1} \frac{\partial}{\partial \zeta} s'(r, \zeta, 0, t) \]  

(A2b)

When the initial condition is taken to be:
\[ s'(r, \zeta, 0) = 0 \]  

(A2c)

the Laplace transform of eqn. (A1') is given by:
\[ \frac{\partial^2}{\partial \zeta^2} \tilde{s}(r, \zeta, p) = pCS' \tilde{s}'(r, \zeta, p) \]  

(A1'')
Equations (A2a, b) remain unaltered under Laplace transformation, except for a bar appearing above \( \phi \) and \( a \). By inspection the solution of eqn. (A1'), using eqn. (A2a), is found to be:

\[
\check{\phi}(r, \zeta, p) = \sinh^{-1}\left(\sqrt{\rho CGS^*}\right) \sinh^{-1}\left(1 - \zeta \sqrt{\rho CGS^*}\right) H^T - \sinh^{-1}\left(\sqrt{\rho CGS^*}\right) \check{a}(r, p) \tag{A3}
\]

Using Darcy's law the downward flux \( i \) through the aquitards is found from eqn. (A3):

\[
i(r, \zeta, p) = -C^{-1} \frac{\partial}{\partial \zeta} \check{\phi}(r, \zeta, p) = -C^{-1} \sinh^{-1}\left(\sqrt{\rho CGS^*}\right) \sqrt{\rho CGS^*} \tag{A4}
\]

With regard to the aquifers the matrix differential equation describing the hydraulic head is:

\[
TV \check{a}(r, \zeta, p) - H \check{i}(r, 0, p) + \check{i}(r, 1, p) = \rho S \check{a}(r, \zeta, p) \tag{A5}
\]

where:

\[
\nabla^2 = \frac{\partial^2}{\partial r^2} + \frac{1}{r} \frac{\partial}{\partial r}
\]

The boundary conditions are:

\[
\lim_{r \to 0} -2\pi r T \frac{\partial}{\partial r} a(r, \zeta, p) = q \tag{A6a}
\]

\[
a(\infty, \zeta, p) = 0 \tag{A6b}
\]

With the initial condition:

\[
a(r, 0, \zeta) = 0 \tag{A6c}
\]

the Laplace transform of eqns. (A5) and (A6a, b) reads:

\[
TV \check{a}(r, p) - H \check{i}(r, 0, p) + \check{i}(r, 1, p) = \rho S \check{a}(r, p) \tag{A5'}
\]

\[
\lim_{r \to 0} -2\pi r T \frac{\partial}{\partial r} \check{a}(r, p) = \frac{1}{p} q \tag{A6a'}
\]

\[
\check{a}(\infty, p) = 0 \tag{A6b'}
\]

In view of eqn. (A4), eqn. (A5') can be written:

\[
TV \check{a}(r, p) + B(p) \check{a}(r, p) = \rho S \check{a}(r, p) \tag{A5''}
\]

where:

\[
B = HC^{-1} \sinh^{-1}\left(\sqrt{\rho CGS^*}\right) \sinh^{-1}\left(1 - \zeta \sqrt{\rho CGS^*}\right) H^T + I
\]

\[
\sinh^{-1}\left(\sqrt{\rho CGS^*}\right) - H^T + \sinh^{-1}\left(\sqrt{\rho CGS^*}\right)
\]

The matrix \( B \) is seen to be symmetrically tridiagonal. In scalar notation:

\[
\{B_{u}\} = \frac{1}{c_{1}} \frac{\sqrt{\rho CGS^*}}{\tanh \sqrt{\rho CGS^*}} - \frac{1}{c_{1}} \frac{\sqrt{\rho CGS^*}}{\tanh \sqrt{\rho CGS^*}}
\]

\[
\{B_{u+1}\} = \frac{1}{c_{1}} \frac{\sqrt{\rho CGS^*}}{\sinh \sqrt{\rho CGS^*}}
\]

\[
\{B_{u+1}\} = \{B_{u+1}\} \tag{A7'}
\]
The general solution of eqn. (A5\textsuperscript{*}) is given by:

\[ \tilde{u}(r, p) = K_a(r, \sqrt{A(p)})c_1 + L_a(r, \sqrt{A(p)})c_2 \]  

(A8)

where:

\[ A(p) = T^{-1}[pS - B(p)] \]

c\text{ and } c\text{ are column matrices containing constants of integration. } c\text{ and } c\text{ are solved by using eqn. (A8) and the boundary conditions (A6\textsuperscript{a}) and (A6\textsuperscript{b}). It is found that:}

\[ c_1 = \frac{1}{2\pi} T^{-1}q \]

(A9a)

\[ c_2 = 0 \]

(A9b)

so the final solution reads:

\[ \tilde{u}(r, p) = \frac{1}{2\pi} K_a(r, \sqrt{A(p)}) T^{-1}q \]  

(A9c)

A finite system of \( n \) aquifers and \( n + 1 \) aquitards can be obtained by setting:

\[ \tilde{u}_n(r, p) = 0 \]  

(A10a)

\[ \tilde{z}_n+1(r, p) = 0 \]  

(A10b)

Alternatively, an impervious base may be introduced by setting:

\[ \{B_m\} = -\frac{1}{c_0} \text{tanh}\sqrt{\frac{pc_x S_y}{c_0\sqrt{S_x}}} \]  

(A11a)

and an impervious top layer is obtained likewise by:

\[ \{B_{11}\} = -\frac{1}{c_0} \text{tanh}\sqrt{\frac{pc_x S_y}{c_0\sqrt{S_x}}} \]  

(A11b)

Numerical evaluation of the matrix function \( K_a(r, \sqrt{A}) \) is performed as described by Maas (1986).

**APPENDIX B. DERIVATION OF EQN. (5) BY EIGENVALUE ANALYSIS**

The application of the Laplace transform to the boundary value problem given by eqns. (1) and (2) yields a system of \( 2n \) ordinary differential equations.

Vertical flow in the aquitards is described by \( n \) equations:

\[ \frac{\partial^2 \tilde{u}_i}{\partial z^2} = \frac{p K_i}{S_i} \tilde{u}_i, \quad i = 1, 2, \ldots, n \]  

(B1)

\[ \tilde{u}_i(r, z_{i-1}, p) = \tilde{z}_{i-1}(r, p) \]

\[ \tilde{u}_i(r, z_i, p) = \tilde{z}_i(r, p) \]

while \( \tilde{z}_0 = \tilde{z}_{n+1} = 0 \) when a leaky system is considered. By substituting \( \tilde{q}_i = pS_z/K_i \) and \( \tilde{b}_i = a_i \tilde{q} \)

the solution obtained can be written:

\[ \tilde{u}_i = \frac{\sinh(\alpha_i z - \alpha_{i-1} z_i)}{\sinh \tilde{b}_i \tilde{b}_{i-1}} \tilde{z}_{i-1} + \frac{\sinh(\alpha_i z_{i-1} - \alpha_{i-1} z)}{\sinh \tilde{b}_{i-1} \tilde{b}_i} \tilde{z}_i \]

from which it follows that:
If, however, the base of aquitard $i$ is impervious, the second boundary condition of eqn. (B1) should be replaced by:

$$\frac{\partial}{\partial z} \tilde{z}(r, z', p) = 0$$

which leads to a solution of the form:

$$\tilde{z}' = \frac{\cosh(a_i z - a_i z')}{\cosh b_i} \tilde{z}_{i-1}$$

and hence:

$$\frac{\partial \tilde{z}}{\partial z} = \frac{a_i \sinh(a_i z - a_i z')}{\cosh b_i} \tilde{z}_{i-1}$$

When, subsequently, the system of differential equations for horizontal flow is considered, eqn. (2), Laplace transformation will yield:

$$\frac{\partial^2 \tilde{z}}{\partial r^2} + \frac{1}{r} \frac{\partial \tilde{z}}{\partial r} + \frac{K_r' \partial \tilde{z}}{T_i \partial z} \left| - \frac{K_{r+1} \partial \tilde{z}}{T_{i+1} \partial (z)} \right| - \frac{p T_i}{T_i} \tilde{z}_i = 0, \quad i = 1, 2, \ldots, n$$

At this stage the system of $n$ equations is readily expressed in matrix notation:

$$L \tilde{z} = A \tilde{z}$$
where \( L \) is the Laplace operator \( \frac{\partial^2}{\partial x^2} + \frac{1}{r} \frac{\partial}{\partial r} \), \( \hat{a} \) is the vector of transformed drawdowns and \( A \) is the tridiagonal \( n \times n \) system matrix, as defined by eqn. (6). From eqns. (B3) and (B4) it follows that systems with no-flow boundaries only require that \( \epsilon_1 \) and (or) \( \epsilon_{n,1} \) of matrix \( A \) are replaced by \((b_1 \tanh b_1/c, T)\) and (or) \((b_{n,1} \tanh b_{n,1})c_n, T\).

Solving eqn. (B5) for \( \hat{a} \) is essentially similar to the steady well flow multiple-aquifer solution (Hemker, 1984). A slightly different approach will be used, however, to fully benefit from the nearly symmetric property of \( A \). We define a symmetric tridiagonal matrix \( D \) and then calculate its \( n \) eigenvalues and eigenvectors:

\[
T^{-1/2}A T^{-1/2} = D = R W R^{-1}
\]

where \( T \) is a diagonal matrix with \( T_i \) along the main diagonal, \( W \) is the \( n \times n \) diagonal matrix with the eigenvalues \( \omega_i \), and \( R \) is the \( n \times n \) matrix containing the corresponding eigenvectors in its columns. Since \( D \) is symmetric, the eigenvectors can be normalized to achieve an orthonormal matrix \( R \), thus \( R^{-1} = \frac{1}{\sqrt{\lambda}} \frac{1}{\sqrt{\lambda}} \). Let matrix \( V \) be defined by \( V = T^{-1/2} R \), then:

\[
V^T = \frac{1}{\sqrt{\lambda}} \frac{1}{\sqrt{\lambda}} T^{-1/2} = \frac{R^T T^{-1/2}}{\sqrt{\lambda}}
\]

Substituting eqns. (B6) and (B7) into (B5) leads to:

\[
L \hat{a} = \sqrt{\lambda} W V^{-1} \hat{a}
\]

This system of differential equations can be uncoupled and solved for the boundary conditions, just like the steady-flow problem, to obtain:

\[
\hat{a} = \frac{1}{\sqrt{\lambda}} \sqrt{\lambda} K V^{-1} g
\]

where \( K \) is the \( n \times n \) diagonal matrix with \( K_i(r, \sqrt{\omega}) \) as non-zero elements and \( g \) is the vector given by \( G_i \sqrt{\omega} T_i; i = 1, 2, \ldots, n \). By further simplification the final solution may be written:

\[
\hat{a} = \frac{1}{2\sqrt{\lambda \pi}} \sqrt{\lambda} K V^{-1} q
\]

where \( q \) is simply the discharge vector \((Q_1, Q_2, \ldots, Q_n)^T\). This last equation shows that its numerical evaluation only implies the eigenvalue decomposition of a symmetric tridiagonal matrix and the calculation of hyperbolic and Bessel functions.

REFERENCES