Analytical Methods for the Prediction of Leachate Plume Migration

by Douglas C. Kent, Wayne A. Pettyjohn and Thomas A. Prickett

Introduction

A recent (1980) project funded by the U.S. Environmental Protection Agency (EPA) (Pettyjohn, Kent, Prickett, LeGrand and Witz, in press) has developed a manual to guide government agencies in evaluating hydrogeologic aspects in permit applications dealing with hazardous waste treatment, storage and disposal facilities, as described in Section 3004 of the Resource Conservation and Recovery Act. The techniques described for predicting leachate movement can also be used to develop monitoring plans, and so would be useful for consultants and companies having hydrogeologists on their staff.

Major Approach Used

The plume evaluation system is quantitative and centers on an analytical model by Wilson and Miller (1978). This model or equation is suitable for computer manipulation and is directed to determining the extent of plume migration under varying conditions. Because of its logically precise and deterministic framework, the method is suitable for critical waste and contamination situations where collection of more accurate information can be economically justified. With less precise input data, the method is conceptually useful in approximating hazard potential. The best input data may have accuracy no closer than one order of magnitude in many cases, but the results may help to visualize whether or not a plume might exist and, if so, to what extent the plume might migrate within a specified time. The understanding of plume migration acquired through the use of these quantitative methods should provide another tool for evaluating an engineering design and monitoring program for a proposed or existing hazardous waste site. Following this evaluation, engineering design and monitoring may be modified to provide safety factors sufficient to mitigate any potential plume movement in the ground water.

Data Requirements and Estimates

In order to collect all the facts needed for analysis, a list of data requirements was designed (Table 1). These data are essential for solutions to the analytical methods for predicting plume migration used in this article.

In any ground water pollution study it is essential to obtain the background concentration of a wide variety of chemical constituents, particularly those that might be common both to the local ground water and a leachate. The water in shallow or surficial aquifers can undergo substantial fluctuations in chemical quality and therefore it is not always simple or easy to determine background concentrations, particularly of the more conservative constituents such as chloride or nitrate. In general, samples should be collected during dry periods and not during a rainfall period or within a week or so following it. Throughout much of North America the major period of ground water recharge occurs in wetter periods of the year (generally in the spring) while minor recharge occurs during or immediately after a rain. These recharge events flush water-soluble compounds from the unsaturated zone to the water table and may substantially change the chemical quality of the ground water (Pettyjohn 1982). Since the quality of shallow ground water may fluctuate within fairly wide limits during short intervals, background concentrations should be determined statistically by collecting several samples at different times and from different depths.

The severity of ground water pollution depends partly on the characteristics of the waste or leachate, that is, its volume, composition, concentration of the various constituents, time rate of release of the contaminant, the size of the source area, and the density of the leachate, among others. Data describing these parameters are difficult to obtain and are lumped together into the term "mass flow rate," which is subsequently used in this article.

Once the leachate is formed it begins to migrate slowly downward through the unsaturated zone where several physical, chemical and biological forces act upon it. Eventually, however, the leachate may reach saturated strata where it will then flow primarily in a horizontal direction as defined by the hydraulic gra-
Dispersive processes. From this point on, the concentration will decrease due to a combination of molecular diffusion, which is important only at very low velocities, and dispersion (hydrodynamic mixing). In porous media, different microscopic velocities and flow paths are expected. Leachate moving along a shorter flow path or at a higher velocity would arrive at an end point sooner than that part following a longer path or at a lower velocity, thus resulting in hydrodynamic dispersion. Dispersion can be both longitudinal and transverse: the net result is a conic form downgradient of a continuous-pollution source. The leachate concentration is less at the margins of the cone and increases toward the source.

Since dispersion is affected by ground water flow velocity and the configuration of the aquifer's pore space, coefficients must be determined experimentally or empirically for a given aquifer. There is considerable confusion regarding the quantification of the dispersion coefficient. Many published values are fitted and ground water constituents or if'sorption occurs. The rate of advance of a contaminant plume can be retarded if there is a reaction between its components and ground water constituents or if sorption occurs. The plume in which sorption and chemical reactions occur generally will expand more slowly and the concentration will be lower than those of an equivalent nonreactive leachate.

Hydrodynamic dispersion affects all solutes equally while sorption and chemical reactions can affect various constituents at different rates. Thus, a leachate source that contains a number of different contaminants can have each solute moving at a different rate. The areal extent of plumes may range within rather wide extremes depending on the local geologic conditions, influences on the hydraulic gradient, such as pumping, ground water velocity, and changes in the time rate of release of contaminants.

The many complex factors controlling leachate movement and the overall behavior of contaminant plumes are difficult to assess in that the final effect represents several factors integrated collectively. Likewise, concentrations for each constituent in a complex waste are difficult to obtain. Therefore, predictions of concentration and plume geometry are best used as estimates, principally to identify whether or not a plume might develop at a proposed site and, if so, where and to what extent it should be monitored.

**Plume Evaluation System**

The methods outlined herein form two different approaches to the prediction of leachate plume migration. In Phase I, a simple transport equation is the basis for three predictive methods, while in Phase II, a number of more sophisticated computer codes are recommended. The latter have appeared in the literature over the past several years and several are well-documented.

The graphical and computer techniques were developed to predict or evaluate the severity of contamination once a leachate has reached the saturated zone. To make results comparable from one method to another, the literature was searched for a single equation that the authors considered to be satisfactory to model plume migration. The equation was obtained from Wilson and Miller (1978) and the techniques in Phase I are based on it:

$$ C = \frac{f m \exp (x/B)}{4\pi n \sqrt{D_x D_y}} w (u, r/B) $$

The Wilson and Miller (1978) analytical model was chosen because it is a simpler model with fewer parameters than the other equations examined. The simpler solution lends itself to developing the nomograph and the user-oriented calculator and TSO computer programs.

The equation and the methods, however, do have significant drawbacks, the most serious being the inability to predict movement through the unsaturated zone and to input natural recharge between source and sink into the techniques. Recharge can be introduced by means of the more advanced models of Phase II.

A generalized flow diagram for leachate plume prediction using the methods described therein are shown in Figure 1.

**Assumptions and Limitations of Analytical Solution Used in Plume Evaluation System**

The Wilson and Miller (1978) equation is used to provide a deterministic analytical solution for computing the concentration distribution in a two-dimensional contaminant plume. Definitions of the terms used in the equation are listed in Table 1. The assumptions on which the analytical solution is based are as follows:

- The ground water flow regime is saturated
- The aquifer is unlimited (infinite) in areal extent
- All aquifer properties are homogeneous and isotropic
- Ground water flow is continuous and uniform in direction and velocity
- There is no dilution of the plume by recharge outside the source area
- The leachate source is a point in plan view
- The leachate is evenly distributed over the vertical dimension of the saturated zone
- The leachate source supplies a constant mass flow rate based on an initial concentration and volume (recharge rate) of leachate reaching the water table at a point directly below the source.

The first assumption is satisfied where a source concentration is initiated at the top of the saturated zone or at the base of the landfill or lagoon. The assumption of homogeneous and isotropic aquifer properties is rarely encountered in the field, but the availability of data requires the use of a homogeneous approximation. The uniform flow velocity in the fourth assumption requires that the water volume from the source is not large enough to significantly affect the
natural ground water gradient. Ground water recharge and the associated dilution of the plume are not considered. This assumption represents a “worst case,” especially in areas of high precipitation and where permeable materials exist near the land surface.

In the theoretical derivation, it is assumed that the leachate enters the ground water directly below a point source. However, in practice the equation can be used for locations far enough from a non-point source so that it appears to be a point source. Wilson and Miller (1978) have developed a relationship based on the size of the source (assuming rectangular dimensions) in order to establish the minimum distance to the source. A large non-point source can also be handled by dividing it into smaller areas. The effect of each sub-area is calculated separately and the resulting concentrations are added.

Methods Used in Phase I

The analytical solution by Wilson and Miller has been modified for use as a (1) graphical solution (nomograph), (2) hand calculator program, and (3) an interactive program for microcomputers and the IBM 370 computer.

These three methods can be used to predict the following: (1) the maximum distance of plume migration for a given concentration threshold or limit; such concentrations might be those that have been established as standards for safe drinking water by EPA, and (2) the maximum leachate concentration that could be reached at a specific distance from the landfill or lagoon. The prediction is not absolute because of the simplifying assumptions and estimates made for the parameters required in the solution. More importantly, however, the prediction is useful for a better understanding of the hydrogeologic setting and contaminant behavior at an existing or proposed waste site. In order to use the techniques, an individual should assess all hydrogeologic factors that are important in site evaluation. Some parameters may not be well understood even within the scientific community, but reasonable estimates for the “worst case” are useful for initial assessment. If data are poorly defined or unavailable, the methods can be used to identify additional data that are needed and the location where those data should be obtained (Figure 1).

An initial concentration and volume (recharge rate) of leachate reaching the water table is estimated for a point directly below the source. Ordinarily the thickness of vertical mixing in the ground water system is assumed to be the entire saturated thickness of the aquifer; however, this can be modified to approximate the effects of layering or density stratification.

Nomograph

A graphical solution was developed to provide a simple computational tool for predicting leachate plume movement and corresponding concentration. It is often necessary to estimate the potential travel distance or length of time required for a plume to migrate some distance in the saturated zone from a point directly below a contaminant source. The concentration of conservative elements in the plume, such as chloride, can be estimated from some selected point in space and time along a flow path that extends directly downgradient from the source.

The nomograph is intended as a rapid means for obtaining an approximate solution. It also aids in understanding the model and using the hand calculator or interactive computer methods. The nomograph is one-dimensional (restricted to a line) whereas the calculator and TSO computer methods are two-dimensional.

The Wilson and Miller (1978) equation was reformulated to introduce scale factors and to provide the basis for the nomograph as follows:

Figure 1. Generalized flow diagram for leachate plume prediction
\[ C = \frac{QC_0}{4Q_D \sqrt{\pi x/X_D}} \exp \left( \frac{x}{X_D} \sqrt{\frac{\gamma - x}{X_D/2}} \right) \text{erfc} \left( \frac{\phi}{2} \right) \]

where:

\[ \phi = \frac{x/X_D - U/T_D}{\sqrt{U/T_D}} \]

Application allows mapping the center-line concentration of the plume, with respect to time, in one direction (x distance) directly downgradient from the source. Dilution-dispersive mixing and retardation parameters are included in the solution. The equation and the nomograph apply to only one chemical constituent, such as chloride or dissolved solids, at a time.

Three scale factors used in the nomograph are ratios with the primary variables x (distance), t (time), and QC (mass flow rate from source) in the forms of x/D, t/D, and QC/QD. The y distance is set to zero. The scale factors are:

\[ X_D = \frac{D_X}{V} \quad T_D = \frac{R_D D_X}{V^2} \quad Q_D = n m \sqrt{D_X D_Y} \]

Two of the three ratios are computed directly and the third is found using the nomograph. The variables are defined in Table 1. The factors provide two conveniences. First, the ratios are dimensionless except for concentration. Secondly, the scale factors combine the constant parameters, which makes it easier to repeat computations of concentration (C) for various positions along the x axis (x) or for different times (t).

The nomograph in Figure 2 is designed to provide a simple technique to estimate one of the following:

Application 1: The concentration (C) at a selected distance (x) and time (t).

Application 2: The distance (x) where a selected concentration (C) will occur at a given time (t).

Application 3: The time (t) when the concentration will reach a selected concentration (C) at a pre-determined location.

As time passes, the concentration in a given area approaches a constant pattern known as steady-state. Results for steady-state conditions can be determined for the first two applications as follows:

Application 1b: The maximum concentration (C) that would occur at a selected distance after a long time period.

Application 2b: The maximum distance (x) at which a selected concentration (C) will occur after a long time period.

![Nomograph for solutions of time, distance and concentration for any point along the principal direction of ground water flow](image)
### Table 1
**Definition of Terms**

<table>
<thead>
<tr>
<th>Variable</th>
<th>Definition</th>
<th>Units</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>Primary Variables:</strong></td>
<td></td>
<td></td>
</tr>
<tr>
<td>C</td>
<td>Concentration of leachate at a specified time and distance</td>
<td>(M/L³)</td>
</tr>
<tr>
<td>X</td>
<td>Distance from source where concentration of leachate is computed. Distance is measured in direction of ground water flow (perpendicular to gradient)</td>
<td>(L)</td>
</tr>
<tr>
<td>y</td>
<td>Transverse distance measured from the centerline of ground water flow (assumed to be zero in the nomograph)</td>
<td>(L)</td>
</tr>
<tr>
<td>t</td>
<td>Sample time from beginning of leachate source flow</td>
<td>(T)</td>
</tr>
<tr>
<td><strong>Aquifer Parameters:</strong></td>
<td></td>
<td></td>
</tr>
<tr>
<td>m</td>
<td>Effective aquifer thickness or zone of mixing</td>
<td>(L)</td>
</tr>
<tr>
<td>n</td>
<td>Effective porosity of aquifer or zone of mixing</td>
<td>(Dimensionless)</td>
</tr>
<tr>
<td>V</td>
<td>Velocity of ground water flow within voids: estimated directly or from: ( V = \frac{KI}{n} )</td>
<td></td>
</tr>
<tr>
<td></td>
<td>where: ( K = ) Coefficient of permeability or hydraulic conductivity of aquifer or zone of mixing; ( I = ) Gradient of ground water flow</td>
<td>(Dimensionless)</td>
</tr>
<tr>
<td><strong>Transport Parameters:</strong></td>
<td></td>
<td></td>
</tr>
<tr>
<td>( D_x )</td>
<td>Longitudinal dispersion coefficient (mixing rate) with respect to distance in x direction and time: estimated directly or from: ( D_x = \alpha_x V + D^* )</td>
<td>(L²/T)</td>
</tr>
<tr>
<td>( D_y )</td>
<td>Transverse dispersion coefficient (mixing rate) with respect to distance in the y direction and time: estimated directly or from: ( D_y = \alpha_y V + D^* )</td>
<td>(L²/T)</td>
</tr>
<tr>
<td>( R_d )</td>
<td>Retardation factor estimated directly or from: ( R_d = 1 + \frac{\rho b (Kd)}{n_t} ) (or) ( R_d = \frac{V}{V_d} )</td>
<td>(Dimensionless)</td>
</tr>
</tbody>
</table>

\( \alpha_x \) = Longitudinal dispersivity

\( \alpha_y \) = Transverse dispersivity

or estimated as:

\( D_y = D_x \) divided by a ratio, which commonly ranges between 5 and 10 for medium to coarse sand aquifers
Table 1
Definition of Terms
(Continued)

where:

\begin{align*}
\rho_b &= \text{Bulk density of aquifer medium} \quad (\text{M/L}^3) \\
n_t &= \text{Total porosity} \quad \text{(Dimensionless)} \\
K_d &= \text{Distribution factor for sorption on aquifer medium (from sorption isotherm column studies)} \quad (\text{L}^3/\text{M}) \\
V &= \text{Velocity of ground water} \quad (\text{L/T}) \\
V_d &= \text{Observed velocity of leachate for a given concentration and chemical species} \quad (\text{L/T}) \\
\gamma &= \text{Coefficient for radioactive or biological decay. For no decay, the value of } \gamma \text{ is one. (Dimensionless)} \\
&= \text{(Assumed to be one in the nomograph.) Calculated from:} \\
&= \frac{4D_x}{\gamma V^2} + \frac{4D_x \log (2)}{V^2 t_{1/2}} \\
&\text{where:} \\
\lambda &= \text{Decay constant} = \frac{\log(2)}{t_{1/2}} \quad (1/\text{T}) \\
t_{1/2} &= \text{Halflife: time when half of the original mass remains} \quad (\text{T})
\end{align*}

Source Rate of Leachate:

\begin{align*}
Q_{C_0} &= \text{Mass flow rate estimated directly or obtained from the product of:} \quad (\text{M}/\text{T}) \\
Q &= \text{Volume flow rate estimated directly or from:} \quad (\text{L}^3/\text{T}) \\
&= Aq \\
&\text{where:} \\
A &= \text{area of source} \quad (\text{L}^2) \\
q &= \text{recharge rate} \quad (\text{L/T}) \\
C_0 &= \text{Initial concentration} \quad (\text{M}/\text{L}^3)
\end{align*}

Intermediate Variables (Used for nomograph only):

\begin{align*}
X_D &= \text{A characteristic dispersion length or scale factor given by:} \quad (\text{L}) \\
X_D &= \frac{D_x}{\sqrt{\gamma V}} \\
T_D &= \text{A characteristic dispersion time or scale factor given by:} \quad (\text{T}) \\
T_D &= \frac{R_d D_x}{\gamma V^2} \\
Q_D &= \text{A characteristic dilution-dispersion flow} \quad (\text{L}^3/\text{T}) \\
Q_D &= n m \sqrt{D_x D_y}
\end{align*}

The advantage of application 1b and 2b is that it is possible to predict, for example, the maximum distance of plume migration for a given concentration threshold or limit. Such concentrations might be those that have been established as standards for the safe drinking water by EPA. Alternatively, it is also possible to predict the maximum leachate concentration that could be reached at a specified distance from the landfill or lagoon.

The estimate of distance (x), time (t) and concentration (C) may require an adjustment of the concentration value to correct for significant background concentrations. In applications 1 and 1b the estimated concentration (C) must be added to the background concentration. In applications 2, 2b and 3 the concentration value used must be the remainder after subtracting the natural background concentration.
Hand Calculator Method

The TI-59 calculator code is also based on the Wilson and Miller (1978) equation. It was developed for quick and convenient solutions for determining the concentration distribution in a leachate plume in simple two-dimensional problems. With proper input data, the calculator code will allow modeling of dispersion, ion-exchange or retardation, radioactive decay and dilution from native ground water passing beneath the landfill from upgradient sources. The calculator model includes the capabilities to analyze a small variable strength leachate source, areally distributed landfill sites, and the plan-view description of plume concentrations under non-steady as well as steady-state conditions.

Computer Program

A program also based on the Wilson and Miller (1978) equation, was developed to calculate plume concentrations by means of larger computers. The program is written in BASIC language and adapted for use either in a microcomputer or an interactive terminal using TSO or the IBM 370. The program can calculate and display the concentration at a single point or as a grid map of concentrations.

As shown by the flowchart in Figure 3, the program operates by requesting a command code from the user, which designates a particular operation to be performed. The codes may be entered in any order, one at a time. Of course, all desired parameters must be set before requesting the concentration calculation. A detailed explanation of the required parameters is given in Table 1.

During program execution all parameters retain their values until changed by the user. When the program is started, each parameter is initialized. If the initial value is the value desired for the first case to be run, then the parameter need not be entered. This is especially useful for retardation and decay, which are initially set to 1 (no retardation or decay).

The basic equation for concentration assumes a constant mass flow rate. However, the equation can be applied to a number of time periods, each having a different mass flow rate. The computer program allows the specification of up to 10 time periods with a starting time and a mass flow rate for each period. The concentration can be calculated for sample times during any time period.

Sensitivity Analysis of Data Used in the Three Methods

The nomograph diagram in Figure 2 provides a visual representation of the plume concentration. The solution can be found easily for various locations, times and concentrations. This leads to gaining a "feel" for the nature of the plume. As time passes, the concentration at a given location reaches a constant value known as steady-state. The steady-state value for concentration can be useful for example as a "worst case" scenario (maximum concentration reached in the infinite time). The upper line on the nomograph represents the time and distance at which steady-state is reached. It is easy to see that before steady-state, small changes in location or time correspond to large changes in concentration. In a sense, the steepness of the non-steady-state time lines indicates that the "leading edge" of the plume is relatively narrow and therefore passes a given location in a relatively short period of time. Behind the leading edge, the concentration remains constant at the steady-state value represented by the steady-state line on the nomograph.

Sensitivity analysis was performed using the TSO computer program for the basic aquifer and transport parameters of velocity (V), dispersion coefficient (Dx), dispersion ratio (Dx/Dy), retardation factor Rm, decay coefficient (γ), aquifer thickness (m), and porosity (n). For each parameter, the concentration at location X = 4,200 feet, y = 0 feet, was calculated for a time of 2,333.3 days and for steady-state (infinite time).

Table 2 lists the results of varying the velocity (V) from 0.015 to 5.0 ft/day. The steady-state concentration is reduced with greater velocity due to increased dilution. For small values of velocity, the non-steady-state concentration is negligible because the contamination has not arrived at the sample point.

Table 3 is used to summarize the sensitivity from the most sensitive parameter to the least sensitive. The mathematical expression listed under "effect" gives the rate of change in concentration. For example, 1/m indicates that increasing the thickness by a factor of two will decrease the concentration by a factor of two. The expression 1/V indicates that increasing velocity by a factor of four will decrease concentration by a factor of two.

Phase II

Hopefully, nearly all new landfill sites can be evaluated and permitted or rejected on the basis of Phase I, and the applicant or permit writer will not have to resort to a complex analysis that requires complicated numerical modeling techniques to make an accep-
Table 2
Concentration as a Function of Velocity (v)

<table>
<thead>
<tr>
<th>Velocity (v)</th>
<th>X_D (ft)</th>
<th>T_D (days)</th>
<th>O_D (ft³/day)</th>
<th>T/T_D time = 2,333.3 days</th>
<th>Concentration at point 4,200 ft. (mg/L)</th>
</tr>
</thead>
<tbody>
<tr>
<td>(ft/day)</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>0.015</td>
<td>7,000.000</td>
<td>466,666.700</td>
<td>1,807.861</td>
<td>0.600</td>
<td>X/Xc</td>
</tr>
<tr>
<td>0.050</td>
<td>2,100.000</td>
<td>42,000.000</td>
<td>—</td>
<td>2.000</td>
<td>—</td>
</tr>
<tr>
<td>0.500</td>
<td>1,050.000</td>
<td>10,500.000</td>
<td>—</td>
<td>4.000</td>
<td>—</td>
</tr>
<tr>
<td>0.150</td>
<td>700.000</td>
<td>4,666.700</td>
<td>—</td>
<td>6.000</td>
<td>—</td>
</tr>
<tr>
<td>0.500</td>
<td>210.000</td>
<td>420.000</td>
<td>—</td>
<td>20.000</td>
<td>22.200</td>
</tr>
<tr>
<td>1.000</td>
<td>105.000</td>
<td>105.000</td>
<td>—</td>
<td>40.000</td>
<td>34.700</td>
</tr>
<tr>
<td>1.250</td>
<td>84.000</td>
<td>67.200</td>
<td>—</td>
<td>50.000</td>
<td>74.990</td>
</tr>
<tr>
<td>1.500</td>
<td>70.000</td>
<td>46.700</td>
<td>—</td>
<td>60.000</td>
<td>50.000</td>
</tr>
<tr>
<td>1.800</td>
<td>58.300</td>
<td>32.400</td>
<td>—</td>
<td>72.000</td>
<td>71.990</td>
</tr>
<tr>
<td>2.000</td>
<td>52.500</td>
<td>26.250</td>
<td>—</td>
<td>80.000</td>
<td>88.890</td>
</tr>
<tr>
<td>2.500</td>
<td>42.000</td>
<td>16.800</td>
<td>—</td>
<td>100.000</td>
<td>138.890</td>
</tr>
<tr>
<td>3.000</td>
<td>35.000</td>
<td>11.670</td>
<td>—</td>
<td>120.000</td>
<td>199.997</td>
</tr>
<tr>
<td>3.500</td>
<td>30.000</td>
<td>8.570</td>
<td>—</td>
<td>140.000</td>
<td>272.218</td>
</tr>
<tr>
<td>4.000</td>
<td>26.300</td>
<td>6.560</td>
<td>—</td>
<td>160.000</td>
<td>355.500</td>
</tr>
<tr>
<td>5.000</td>
<td>21.000</td>
<td>4.200</td>
<td>—</td>
<td>200.000</td>
<td>555.500</td>
</tr>
</tbody>
</table>

Table 3
Effect of Increasing the Value of a Parameter on Concentration

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Steady-State Effect</th>
<th>Non-Steady-State Effect</th>
</tr>
</thead>
<tbody>
<tr>
<td>decay coefficient (γ)</td>
<td>very large decrease</td>
<td>very large decrease</td>
</tr>
<tr>
<td>aquifer thickness (m)</td>
<td>decrease as $\frac{1}{m}$</td>
<td>decrease as $\frac{1}{m}$</td>
</tr>
<tr>
<td>porosity (n)</td>
<td>decrease as $\frac{1}{n}$</td>
<td>decrease as $\frac{1}{n}$</td>
</tr>
<tr>
<td>velocity (V)</td>
<td>decrease as $\sqrt{v}$</td>
<td>large increase at leading edge</td>
</tr>
<tr>
<td>dispersion coefficient (D_y)</td>
<td>decrease as $\frac{1}{\sqrt{D_y}}$</td>
<td>moderate increase at leading edge</td>
</tr>
<tr>
<td>dispersion ratio (D_y/D_x)</td>
<td>decrease as $\frac{D_x}{D_y}$</td>
<td>increase as $\frac{D_x}{D_y}$</td>
</tr>
<tr>
<td>retardation factor (R_d)</td>
<td>no change</td>
<td>very large decrease at leading edge</td>
</tr>
</tbody>
</table>
Applications for Phase I
Hydrologic Setting

The groundwater contamination case used in this example occurred in South Farmingdale, Nassau County, New York, and was described by Perlmutter and Lieber (1970). Most of the data required for the solution as described herein were obtained directly from their report.

Contamination was caused by cadmium and hexavalent chromium-enriched electroplating wastes that infiltrated from disposal basins into a shallow glacial aquifer. Apparently disposal began in 1941 and continued intermittently for several years. By the early 1960s a leachate plume originating at the disposal ponds extended downgradient about 4,300 feet, and was as much as 1,000 feet wide and 70 feet thick. The plume extended to the headwaters of Massapequa Creek, a small stream that serves as a natural drain for part of the contaminated water (Figure 4).

The surficial or upper glacial deposits, which are Pleistocene in age, extend from land surface to a depth of 80 to 140 feet and lie on the Magothy Formation, a unit of aluvial deposits of Late Cretaceous age. The water table in the surficial deposits lies from 0 to about 15 feet below land surface.

The aquifer is in dynamic equilibrium and receives about 22 inches or about 1 mgd (million gallons per day) per square mile of recharge from precipitation. The water table gradient averages about 1 foot in 500 feet and the water table undergoes an annual fluctuation of 2 to 3 feet. The direction of ground water flow is southward from the disposal ponds toward Massapequa Creek. Calculations of ground water velocity for the area range from 0.5 to 1.5 feet per day. The average velocity for the area is reported to be 1 foot per day.

Chemical analyses of ground water in the South Farmingdale area indicate that the background concentration of hexavalent chromium is less than 0.01 mg/L. Likewise, the concentration is also less than 0.01 mg/L in Massapequa Creek upstream of the leachate plume. Along that stretch where the plume discharges into the stream, the chromium concentration is substantially greater.

During the early 1940s, as much as 200,000 to 300,000 gallons per day of effluent (equivalent to 52 pounds per day of chromium) were discharged into three disposal pits having a combined area of about 15,470 square feet. After 1945 the volume of the waste stream was reduced substantially and eventually a treatment plant was constructed. On two occasions the chromium concentration in the raw effluent was 28 and 29 mg/L.

The relatively clean nature (free of clay or organic matter) of the materials forming the surficial aquifer precluded any significant reduction in the chromium load in the plume. That is, ion-exchange during movement was negligible. Maximum chromium concentration in the plume ranged from about 40 mg/L in 1949 to about 10 mg/L in 1962.

The plume is about 200 feet wide at its origin at the disposal ponds. It reaches a maximum length of about 4,300 feet and increases in width to about 1,000 feet. Assuming a velocity of 1.5 feet per day this plume has an average longitudinal dispersion (D_x) of 105 ft^2/day, a transverse dispersion (D_y) of 21 ft^2/day, and dispersivities of 70 feet (a_x) and 14 feet (a_y), respectively. A summary of the required data is shown in Table 4.

In Table 4, Application of the three methods to this example is given in the following sections.

Nomograph Solution
Application 1 (Figure 5)

To find concentration (C) for a distance (x) of 4,200 feet from the source and time (t) of 2,300 days, calculate:

\[
\frac{x}{x_0} = \frac{4,200 \text{ ft}}{70 \text{ ft}} = 60 \text{ Locate at (A)}
\]

\[
\frac{t}{T_D} = \frac{2,300 \text{ days}}{46.7 \text{ days}} = 50 \text{ Locate curve (E)}
\]

\[
\frac{Q C_0}{Q D} = \frac{(26,800 \text{ ft}^3/\text{day}) (31 \text{ mg/L})}{1,800 \text{ ft}^3/\text{day}} = 460 \text{ mg/L Locate at (D)}
\]

Table 4
Summary of Data for Example 1

<table>
<thead>
<tr>
<th>parameter</th>
<th>value</th>
</tr>
</thead>
<tbody>
<tr>
<td>thickness: m</td>
<td>110 ft</td>
</tr>
<tr>
<td>porosity: n</td>
<td>0.35</td>
</tr>
<tr>
<td>velocity: V</td>
<td>1.5 ft/day</td>
</tr>
<tr>
<td>dispersion: D_x</td>
<td>105 ft^2/day</td>
</tr>
<tr>
<td>D_y</td>
<td>21 ft^2/day</td>
</tr>
<tr>
<td>retardation: R_d</td>
<td>1</td>
</tr>
<tr>
<td>volume flow rate: Q</td>
<td>26,800 ft^3/day</td>
</tr>
<tr>
<td>source concentration: C_o</td>
<td>31 mg/L</td>
</tr>
<tr>
<td>mass flow rate: QC_o</td>
<td>26,800 ft^3/day x 31 mg/L or 52 lb/day</td>
</tr>
</tbody>
</table>
Figure 5. Applications la and lb: using the nomograph to estimate the concentration given values of distance and time

or:

\[ \frac{Q_{C_0}}{Q_D} = \frac{52 \text{ lb/day}}{1,800 \text{ ft}^3/\text{day}} = 0.029 \text{ lb/ft}^3 \]

Locate at (D)

Using Figure 5, draw a line vertically from (A) to the intersection with the \( t/T_D \) curve (B), then horizontally from (B) to (C) and from (C) through the scale (D) to (E), giving \( C = 2.6 \text{ mg/L} \).

Application 1b (Figure 5)

To find the maximum concentration for a given distance for large time, use the steady-state line instead of curve \( t/T_D \). Proceeding as above, a concentration \( C \) of 20 mg/L is read at F.

Application 2 (Figure 6)

To find distance \( x \) where a concentration \( C \) of 2.6 mg/L will occur at a time \( t \) of 2,300 days, calculate:

\[ \frac{x}{X_D} = 60 \]

\[ \frac{X_D}{X_D} = 60 \text{ Locate at (D)} \]

Multiply by \( X_D \) to determine distance \( x \):

\[ x = \left( \frac{x}{X_D} \right) (X_D) = (60) (70 \text{ ft}) = 4,200 \text{ feet} \]

Application 2b (Figure 6)

To find the maximum distance at which a selected concentration will reach a given value for large time, use the steady-state line instead of curve \( t/T_D \) in Figure 6.

Application 3 (Figure 7)

To find time \( t \) when the concentration \( C_I \) will reach 2.6 mg/L at location \( x \) of 4,200 feet, calculate:

\[ \frac{x}{X_D} = 60 \text{ Locate at (D)} \]

\[ \frac{X_D}{X_D} = 60 \text{ Locate at (D)} \]

\[ \frac{Q_{C_0}}{Q_D} = \frac{(26,800 \text{ ft}^3/\text{day}) (31 \text{ mg/L})}{1,800 \text{ ft}^3/\text{day}} = 460 \text{ mg/L Locate at (B)} \]

Using Figure 6, locate the selected concentration at (A). Draw a line from (A) through (B) to (C), then horizontally from (C) to curve (C), and vertically to (E), giving:
Using Figure 7, locate the selected concentration at (A), draw a line from (A) through (B) to (C), then horizontally from (C) to an intersection with a vertical line from (D), giving at (E):

\[ t = \frac{t_D}{T_D} = 50 \]

Multiply by \( T_D \) to determine time (t):

\[ t = \left( \frac{t}{T_D} \right) (T_D) = (50) (46.7 \text{ days}) = 2,300 \text{ days} \]

If the lines intersect above the steady-state line, the concentration will not reach the given value at that location.

**Hand Calculator Method**

Using the data in Table 4, the calculator method can be used to determine the same values shown in the previous example. The calculator has a major advantage over the nomograph solution in that it can be used to determine the concentration distribution throughout a plume by calculating the concentration at a sufficient number of \( x \) and \( y \) coordinates. Like the nomograph, this method can easily be applied in the field in contrast to the microcomputer-TSO option.

The TI-59 calculator is loaded with the program, the data are entered and then the program is executed. The concentration will appear on the display and, if available, a printer. An example of the printer output is shown in Figure 8, which reflects a distance from source of 4,200 feet and a time interval of 10,000 days. The other input data are listed in Table 4. The concentration is shown in line C (16.79 mg/L).

**Interactive (TSO) Computer Method**

Using the same information listed in Table 4, two-dimensional maps can be produced by the interactive computer program as shown in Figure 9. Adjustments in dispersion ratios can be used to calibrate the model from an existing landfill of similar waste and hydrogeologic characteristics. Special considerations are required for very small velocities near a large source. Cases such as these require the use of the computer (or calculator) version.

**Summary**

The methods described herein can be used to estimate or predict what the concentration in a leachate plume might be at some point in space and time. The value or accuracy of the prediction, at best, can be no better than the estimate of the hydrogeologic and chemical parameters that are factored into the equations. Because these parameters can range within wide limits, so can the prediction. Furthermore, the prediction must also conform to fundamental hydrogeologic principles and common sense. The results of these predictive techniques must not be allowed to take
precedence over sound field investigation, data collection and interpretation at potential waste disposal sites. Rather, results should be reflected in a more objective data collection strategy and in the engineering practices proposed for a potential site.

Programs are available from the following source:
Mike Roulier, Project Officer
Environmental Protection Agency
Municipal Environment Research Laboratory
Cincinnati, OH 45268

Similar programs have also been developed for microcomputers (Apple, Kaypro, and IBM-PC) subsequent to this project, and are available from the authors.

References

Figure 8. Printer output for hand-held calculator

| 3. | 0042918 | 01 | b |
| 1. | 6791569 | 01 | c |
| 1. | 6791569 | 01 | Σ |

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Figure 9. TSO grid map of concentration using adjusted dispersion ratio

Biographical Sketches

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